12.4 Displacement and Velocity

In the study of motion, it is sometimes important to know the direction so that you can describe the position of a place in relation to a reference point. For example, Kamloops is located 300 km northeast of Nanaimo. In some situations, it is necessary to know not only the speed at which a vehicle is moving, but also the direction. For example, the Thalys high-speed train travels from Brussels, Belgium southwest to Paris, France at 300 km/h. Both displacement and velocity are quantities that require direction.

STUDY TIP

There are many new vocabulary words in this section. As you read, make a study card for each term. You can use these cards later to study for a chapter exam.

Displacement

Distance is a scalar quantity. A **scalar quantity** has a number and a unit. A quantity that has both a number and a unit is called a magnitude. The distance to an object tells you how far away it is, but does not indicate the direction. For example, a student's home is a distance of 500 m from the school. Figure 1 shows a map of the student's home in relation to the school. You can see that the student's home is actually 500 m north of the school. This is known as the displacement of the student's home from the school.

The **displacement** of an object is defined as the change in position of the object. Displacement is a vector quantity. A **vector quantity** has both magnitude and direction. We use the letter d to represent both distance and displacement. To distinguish between the two quantities, we put a small arrow over the vector symbol. An example of each is shown below.

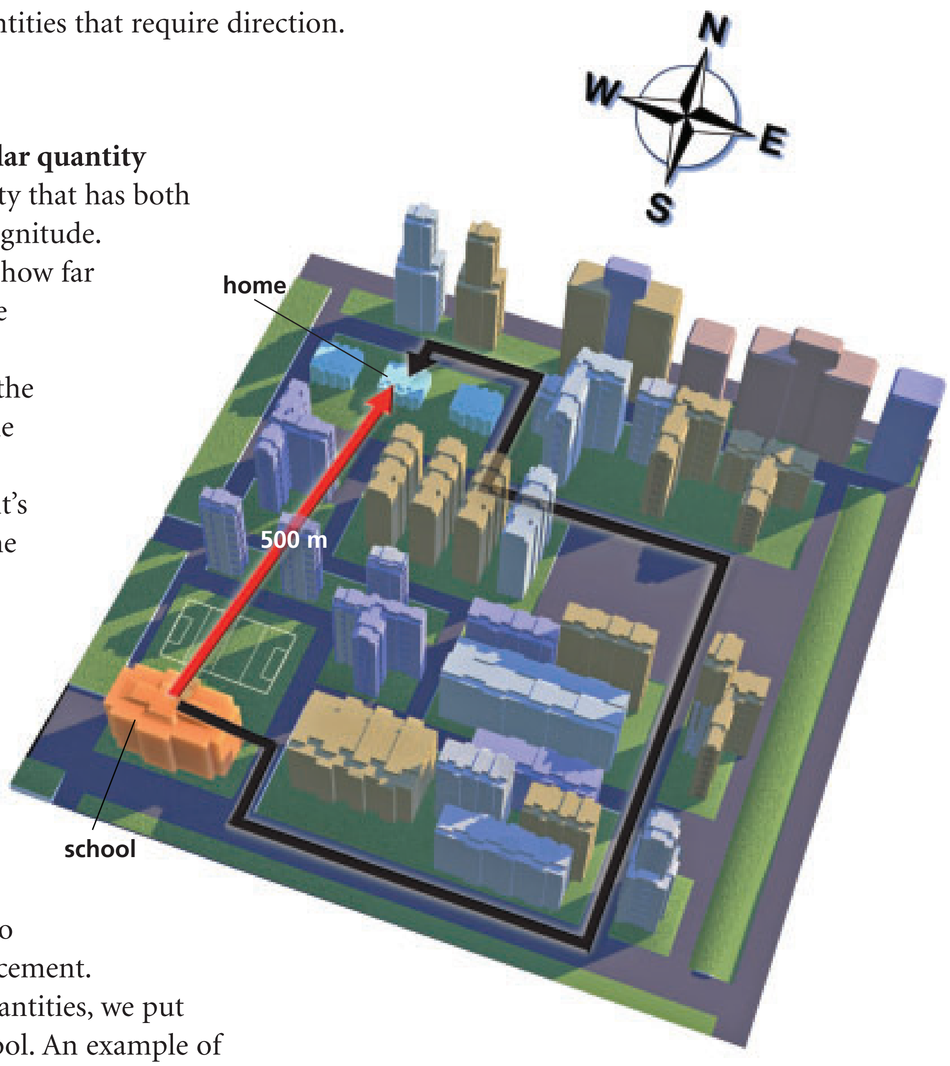

Figure 1 The student's home is 500 m [N] from the school.

$$\text{distance} = \Delta d = 1.5 \text{ km}$$
$$\text{displacement} = \Delta \vec{d} = 312 \text{ m [E]}$$

Note that the symbol for displacement is $\Delta \vec{d}$, which means "change in position."

To calculate the displacement of an object, we need to know its position relative to a reference point. We calculate a change in position by subtracting the initial position, $\vec{d}_i$, from the final position, $\vec{d}_f$. For example, look at Figure 2(a).

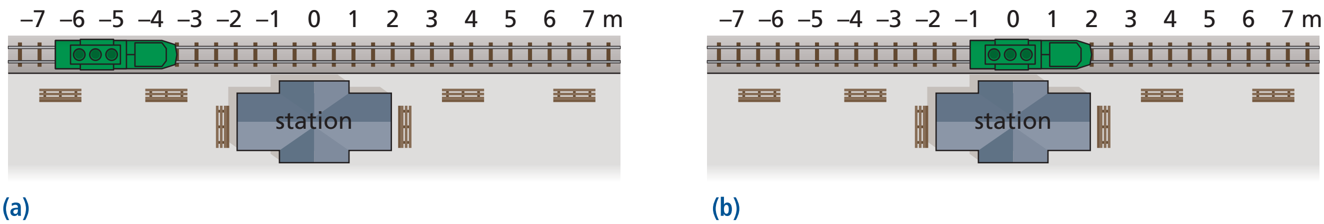

Figure 2 (a) A train is 3.5 units to the left of the train station. We can also indicate the direction by saying −3.5 units. (b) The train moves so that it is 2.0 units to the right of the train station. We can also indicate the direction as +2.0 units.

If the train moves from its position 3.5 units left of the station as shown in Figure 2(a) to a position of 2.0 units right of the station as shown in Figure 2(b), then its displacement from its original position can be calculated as

$$\Delta\vec{d} = \vec{d}_f - \vec{d}_i = 2.0\text{ m} - (-3.5\text{ m}) = 5.5\text{ m}$$

This means that the displacement of the train was 5.5 m to the right.

Direction is important when determining displacement. We calculate changes in position by subtracting the initial position from the final position. In addition, all positions must be relative to the same point. For example, in Figure 2, all positions of the train are relative to the train station. Directions can be left/right, forward/backward, north/south, east/west, or up/down. Remember that we choose one direction to be the reference direction, which will be a positive number, and that the opposite direction will be a negative number. For example, if up is the positive direction, then down will be negative. In Figure 2, right of the train station was positive and left was negative.

SAMPLE PROBLEM 1

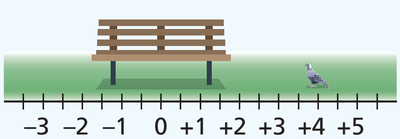

Figure 3 A pigeon stands 4 m right of a bench.

Determine Displacement

A pigeon standing 4 m to the right of a bench walks (Figure 3) to a position 2.5 m to the left of the bench. What is the displacement of the pigeon?

Solution

Let the direction right be positive. Substitute the values into the displacement equation.

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i \\ &= -2.5\text{ m} - (+4.0\text{ m}) \\ \Delta\vec{d} &= -6.5\text{ m}\end{aligned}$$

The displacement of the pigeon is −6.5 m, or 6.5 m left.

Practice

A dog is sitting 1.5 m to the left of a bench. The dog walks so that it is 3.5 m right of the bench. What was the dog's displacement?

Velocity

Speed is the rate of change of distance. As with distance, speed (v) is a scalar quantity because it only has magnitude. **Velocity** ($\vec{v}$) is the rate of change of displacement. Velocity is a vector quantity because it has a magnitude and a direction. The symbol has an arrow over it, in the same way that displacement ($\vec{d}$) does, to indicate that it is a vector quantity. The differences in the equations of speed and velocity are shown below.

$$\text{speed} = v = \frac{\text{distance}}{\text{time}} = \frac{\Delta d}{\Delta t}$$

$$\text{velocity} = \vec{v} = \frac{\text{displacement}}{\text{time}} = \frac{\Delta \vec{d}}{\Delta t}$$

Distance and speed depend on the path taken. Displacement and velocity only depend on the initial and final positions, not the path taken. The following examples illustrate this point. GO

To learn more about the concepts of scalar and vector, go to
www.science.nelson.com GO

SAMPLE PROBLEM 2

Determine Average Speed and Average Velocity

A cyclist trains on the circular track shown in Figure 4. The track has a radius of 100. m and the circumference is 628 m. The cyclist goes 3.5 times around the track in 91.6 s.

(a) What was the average speed of the cyclist?

(b) What was the average velocity of the cyclist?

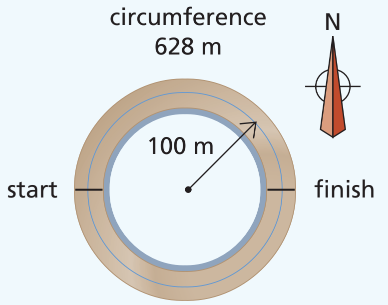

Figure 4 A circular bicycle training track

Solutions

(a) Substitute the values into the average speed equation.

$$v_{av} = \frac{\Delta d}{\Delta t}$$
$$= \frac{3.5\,(628\text{ m})}{91.6\text{ s}}$$
$$v_{av} = 24.0\text{ m/s}$$

The average speed of the cyclist was 24.0 m/s.

(b) First, find the displacement of the cyclist.

$$\Delta\vec{d} = \vec{d}_f - \vec{d}_i$$
$$= 200.\text{ m [E]} - 0\text{ m [E]}$$
$$\Delta\vec{d} = 200.\text{ m [E]}$$

Now, substitute the values into the average velocity equation.

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$
$$= \frac{200.\text{ m [E]}}{91.6\text{ s}}$$
$$\vec{v}_{av} = 2.18\text{ m/s [E]}$$

The average velocity of the cyclist was 2.18 m/s [E].

Practice

A swimming pool is 50 m long. A swimmer completes 150 m in a time of 83 s. The swimmer begins at the south end of the pool and finishes at the north end.

(a) What was the average speed of the swimmer?

(b) What was the average velocity of the swimmer?

Graphing Position and Velocity

Positions can be plotted on a graph in the same way that distances can be plotted. Since the displacement of an object is equal to the change in position of an object, a position-time graph can be used to determine displacements. While the slope of the line of a distance–time graph is equal to the speed, the slope of the line of a position–time graph is equal to the velocity of the object. The major difference is that displacements have a direction.

Table 1 shows the positions of a hiker going on a walk. A graph of the data is shown in Figure 5.

Table 1 Position of a Hiker

Time (min)	Position (m south)
0	0
5	275
10	615
15	865
20	1200
25	1200
30	1200
35	720
40	450

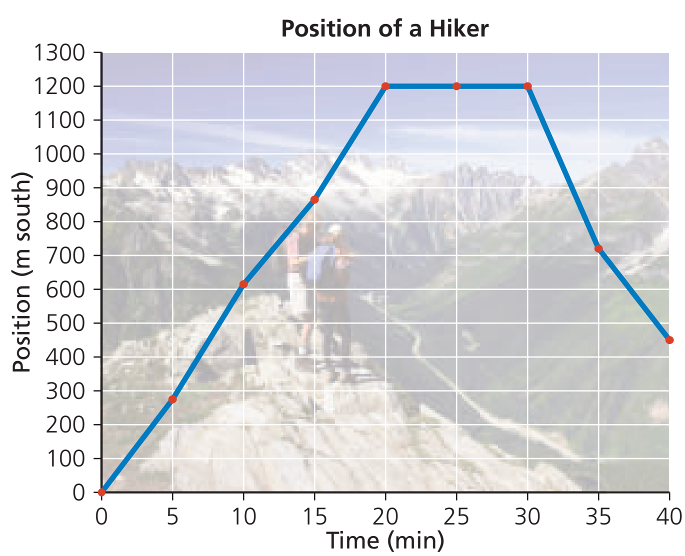

Figure 5 Position–time graph

We can see from the graph that after 20 min, the position of the hiker is 1200 m south. We can calculate the average velocity of the hiker for that time using the equation

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{1200 \text{ m [S]}}{20 \text{ min}} = 60 \text{ m/min [S]}$$

This is an average velocity of 60 m/min [S].

While speeds are always positive, velocities, because they are vector quantities, can be positive or negative with reference to a given direction. For example, the slope of the line between 30 and 35 min is –100 m/min or 100 m/min north.

A car travelling down a twisty road at 60 km/h has a constant speed. However, since the direction of the car is constantly changing, the velocity is constantly changing. If an object is travelling at a constant speed in a constant direction, then the object has constant velocity. This is known as **uniform motion**. A child riding a merry-go-round is travelling at constant speed. However, the child does not have uniform motion because the direction of motion is constantly changing.

Although vectors have both magnitude and direction, sometimes we only need part of a vector quantity. For example, we may only need to know the direction of the displacement from the origin.

To learn more about uniform motion, go to **www.science.nelson.com**

SAMPLE PROBLEM 3

Determine the Magnitude of Average Velocity

A bird flies 300 m [S] in 43 s, lands on a tree branch, and sits for 28 s. Then, the bird turns and flies north 500 m in 62 s. Which of the following is the magnitude of the velocity of the bird?

A. 1.50 m/s

B. 1.90 m/s

C. 6.00 m/s

D. 7.60 m/s

Solution

Let the direction south be positive. The initial position is 0 m. We know that the final position is 300 m + (−500 m) = −200 m, which is the same as 200 m [N]. Substitute the values into the velocity equation.

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{\vec{d}_f - \vec{d}_i}{\Delta t} = \frac{-200.\text{ m} - 0\text{ m}}{43\text{ s} + 28\text{ s} + 62\text{ s}}$$

$$\vec{v}_{av} = -1.50\text{ m/s}$$

Therefore, the velocity of the bird is −1.5 m/s, or 1.5 m/s [N]. However, since we only need the magnitude of the velocity, we do not need to include the direction. The best answer is A. 1.50 m/s.

Practice

A car travels east at 50 km/h and travels 100 km in 2 h. The driver stops for 1 h to have lunch. The driver then continues to travel east 50 km in 1 h. What is the magnitude of the average velocity of the car for this trip?

A. 12 km/h

B. 38 km/h

C. 50 km/h

D. 75 km/h

12.4 CHECK YOUR Understanding

1. Write a definition of magnitude in your own words.
2. Which of the following are vector quantities?
   (a) displacement
   (b) speed
   (c) time
   (d) velocity
3. (a) What is the difference between position and distance?
   (b) What is the difference between position and displacement?
4. Give two examples each of speed and velocity. Use your examples to explain the difference between speed and velocity.
5. A bicycle messenger rides a bicycle around a square city block that has sides that are 100 m long (Figure 6). The messenger begins the ride at corner A.
   (a) When the messenger reaches corner C, what is the distance and the displacement?
   (b) When the messenger returns back to corner A, what is the distance and displacement?

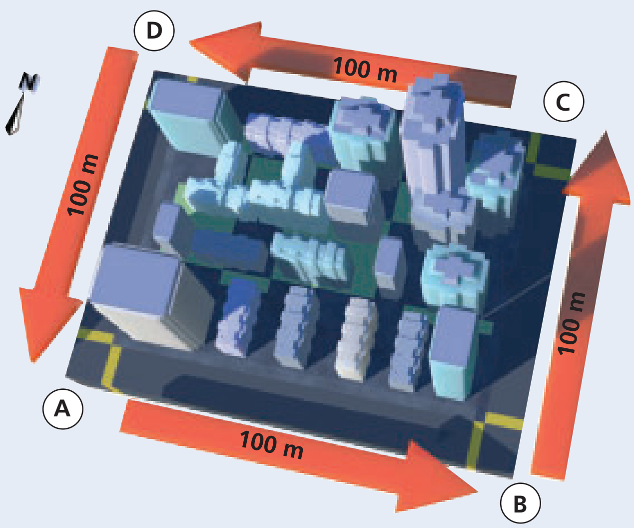

Figure 6

6. A turtle moves 3.5 m [E] in 136 s and then moves 1.7 m [W] in 88 s.
   (a) What is the average speed of the turtle?
   (b) What is the average velocity of the turtle?
7. A cyclist rode a bicycle for a little over 4 min. Her positions were recorded, and a position–time graph for the cyclist is shown in Figure 7.

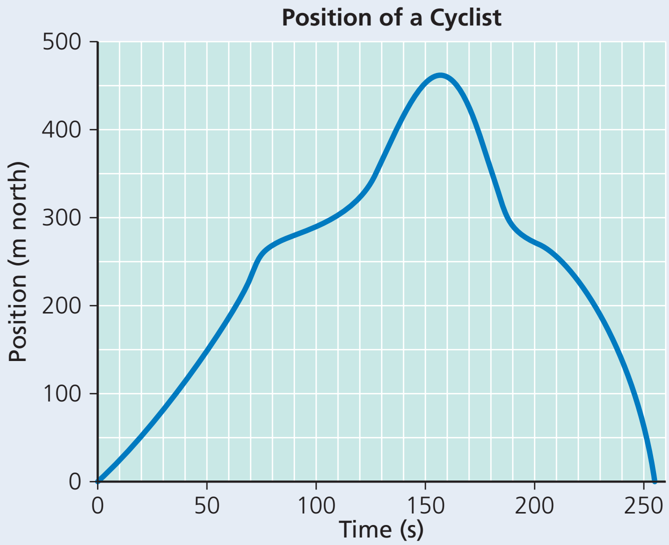

Figure 7

   (a) When was the cyclist's velocity the greatest?
   (b) When was the cyclist's speed the greatest?
   (c) When was the cyclist's position 300 m north?
   (d) What was the cyclist's average velocity for the first 70 s?
   (e) What was the cyclist's average speed for the entire trip?
   (f) What was the cyclist's instantaneous velocity at 240 s?
   (g) Did the cyclist stop at any time during the ride? If so, at what time?
   (h) What was the cyclist's average velocity for the first 200 s?
   (i) What was the cyclist's average velocity for the entire trip?